Single Event Burnout in DC-DC converters for the LHC experiments

C. Rivetta, B. Allongue, G. Berger, F. Faccio, W. Hajdas

Abstract—High voltage transistors in DC-DC converters are prone to catastrophic Single Event Burnout in the LHC radiation environment. This paper presents a systematic methodology to analyze single event effects sensitivity in converters and proposes solutions based on de-rating input voltage and output current or voltage.

I. INTRODUCTION

Experiments for the Large Hadron Collider (LHC), the new particle accelerator under construction at the European Center for Nuclear Research (CERN), are facing new challenges in the design of electronics systems. As an example, all the electronics located inside and around the LHC detectors has to operate reliably in a radiation environment for the LHC lifetime at least 10 years. Due to the huge size of the detectors and of the experimental halls where they are located, power supply systems are preferentially positioned around the detector and as close as possible (between 5 and 10 m) to the front-end electronics. This is a region still exposed to relatively high particle fluxes. In particular, the abundance of high-energy neutrons is a serious threat to the reliable operation of high-voltage power devices. The neutron fluence (above an energy of 20 MeV) over the foreseen 10 years lifetime of the LHC has been estimated, with Monte-Carlo simulations, to be about $1.7 - 3.4 \times 10^8$ n/cm$^2$. The neutron spectrum extends to the GeV region, and peaks at about 60-100 MeV [1]. Total Ionizing Dose (TID) is instead not an issue, at least for the Compact Muon Solenoid (CMS) detector for which this work has been performed. In fact, TID levels are foreseen to be negligible in the periphery of the detector where power supplies will be positioned. The estimated levels are typically well below 1 krad (SiO$_2$), very low even for commercial-off-the-shelf (COTS) components.

A proposed topology for direct-current (DC) low-voltage power distribution consists of AC-DC converters located in the control room that rectify the three phase mains and generate a primary DC voltage of about 200-300V. Each rectifier supplies several DC-DC converters located in the detector-hall near the front-end electronics. Switching regulators then convert the high voltage into appropriate low voltages that are locally distributed to the detector read-out electronics. One family of DC-DC converters fulfilling the electrical specifications for this application is produced and commercialized by VICOR [2]. The salient characteristics of these units are a compact design, low cost, high efficiency and wide variety of input and output voltage and power capability.

The DC-DC converters are COTS components that have to operate reliably in the neutron environment described above; therefore they have to be tested to validate their operation and ensure their reliability in a representative radiation environment. This validation is performed using high-energy proton beams, which are considered equivalent to neutrons inducing single event effects (SEE) at any energy above 20 MeV [3]. The goal of this work is to define appropriate de-rating factors for the input and output variables of the DC-DC converter for reliable operation in the described radiation environment.

In our work, we followed the following steps:
- Analysis of the converter and identification of the component most sensitive to radiation effects leading to part failure
- Characterization of the radiation response of that component (as a function of the voltage bias conditions)
- Determination of the relationship between the operational conditions of the converter (output variables) and the voltage bias conditions of the critical component

Combining all the information obtained, we could define de-rating factors to be applied for reliable operation of the part. Destructive tests using proton beams were then conducted to validate the estimated de-rating factors. This methodology can be extended to validate any power supply operating in a radiation environment dominated by high-energy nucleons.

II. THE VICOR DC-DC CONVERTER

A. Generalities

The VICOR converter is a Forward Quasi-Resonant converter with secondary-side resonance operating in half-wave mode [4]. Figure 1 depicts the schematic diagram of such topology.
The converter transfers energy from the primary side to the secondary side when the switching transistor Q is turned ON during an approximated constant period of time $T_{on}$. This energy is coupled through a resonant circuit composed of the inherent series parasitic inductance of the transformer $L_r$ and the capacitor $C_r$ in the secondary. Figure 2 shows the most important converter waveforms. During $T_{on}$, the switch current $i_l$ follows approximately a half-sinusoidal wave defined mainly by the elements $L_r$ and $C_r$. This current starts from zero when the transistor Q is turned ON describing a positive half cycle that ends when the current $i_l$ turns negative, cutting-off the diode $D_1$. The energy transferred is stored in the resonant capacitor, the output filter and consumed by the load circuit. At the end of the time interval $T_{off}$, the switching transistor Q is turned OFF when current is still flowing through it. This magnetizing current and the magnetic energy still stored in the transformer cannot be reduced to zero too quickly without generating over-voltages. To avoid them, the third coil in the transformer and the reset circuit allows this magnetic energy to be discharged when the switching transistor Q is in the OFF state.

During the time $T_{off}$, the switching transistor is turned OFF and the primary circuit is disconnected from the secondary side. In this interval, only the energy stored in both the resonant capacitor and the output filter is consumed by the load. The complete process is repeated cyclically with a period $T = T_{on} + T_{off}$. The converter output voltage is regulated by balancing the primary energy, transferred when the transistor Q in ON, with the power consumed by the load. The circuit regulates the output voltage via a feedback circuit that adjusts the $T_{off}$ duration. This time interval $T_{off}$ depends mainly on external factors such as the input voltage, output voltage and the load condition. The time $T_{off}$ can change between 500nsec to 10usec depending upon the operating conditions of the DC-DC converter, while the time $T_{on}$ is about 500-600nsec.

B. Characteristic of the converters tested

The converters used during these tests belong to the VICOR family known as MINI. The total power transferred by those converters is in the range of 200-250W. Table I lists the nominal characteristics of the converters. For our application are necessary two converters to deliver to the front-end electronics 7.5V/20A and 5.2V/25A.

<table>
<thead>
<tr>
<th>Converter model</th>
<th>Nominal Vin</th>
<th>Vin Range</th>
<th>Vout</th>
<th>Nominal Iout</th>
</tr>
</thead>
<tbody>
<tr>
<td>V300B5C200A</td>
<td>300V</td>
<td>180V-375V</td>
<td>5V</td>
<td>40A</td>
</tr>
<tr>
<td>V300B12C250AL</td>
<td>300V</td>
<td>180V-375V</td>
<td>12V</td>
<td>21A</td>
</tr>
<tr>
<td>V375B5C200A</td>
<td>375V</td>
<td>250V-425V</td>
<td>5V</td>
<td>40A</td>
</tr>
<tr>
<td>V375B12C250AL</td>
<td>375V</td>
<td>250V-425V</td>
<td>12V</td>
<td>21A</td>
</tr>
</tbody>
</table>

C. Effect of high-energy neutrons on the converter

When exposed to high-energy neutrons, high voltage devices in power converters are susceptible to Single Event Burnout (SEB) and insulated gate power devices are also prone to Single Event Gate Rupture (SEGR). Since SEB is highly dependent on the voltage that the device has to block when it is turned OFF [5][6], the sensitivity of the converter to this effect strongly depends on the external conditions, such as input voltage, output voltage and output current. On the other hand, SEGR is instead mainly dependent on
parameters that are not affected by the magnitude of the external variables of the converter [7][8]. In VICOR stepdown converters, it is possible to predict that the critical components to SEB and SEGR is the switching transistor Q. The drain-source voltage (Vds) of this transistor cannot be measured directly because the converter is sealed into the package by a thermal compound. This voltage can nevertheless be estimated for the switching transistor Q on the basis of the conditions imposed by the transformer.

In addition to SEB and SEGR on power devices, high energy neutrons can induce single event effects (SEE) on the control circuitry which is implemented in an integrated circuit using bipolar technology. Internal devices of this circuit can be affected by single event transients (SET). As a consequence the circuit may induce destructive misfiring on power transistors, transient output voltage dropout, temporary disabling, etc.

D. Estimation of the drain-source voltage of the switching transistor

Analyzing the voltage waveforms from figure 2, during the interval Ton when Q is conducting, the voltage \( V_{ds_{on}} = 0 \) and the voltage across the transformer primary coil \( V_{on} = V_{in} \). During Toff, the voltage across the transistor is \( V_{ds_{off}} = V_{off} + V_{in} \), where \( V_{off} \) is the reflected equivalent voltage in the primary coil due to the reset circuit. The transformer magnetic circuit imposes the condition that in steady-state the time average voltage across any transformer coil should be approximately zero. Assuming a square wave voltage across the transformer primary coil, the voltage magnitude must satisfy the boundary condition:

\[
V_{on} \cdot Ton - V_{off} \cdot Toff = 0 \tag{1}
\]

Inserting this boundary condition into \( V_{ds_{off}} = V_{off} + V_{in} \) and taking into account that \( V_{on} = V_{in} \), we obtain

\[
V_{ds_{off}} = V_{in} \cdot (Ton + Toff) / Toff \tag{2}
\]

This expression allows one to calculate how the voltage drop across switching transistor Q in the OFF condition depends on the input voltage and on the Toff value, which also depends on the load conditions.

The relation between the time Toff and both the output voltage and current can be analyzed considering the balance between the energy transferred from the primary side during the interval Ton and the energy consumed by the load in a period T. In appendix I, the mathematical relation between \( V_{ds_{off}} \) and the output conditions is presented. From this analysis it is possible to establish that \( V_{ds_{off}} \) increases almost linearly with the output current and output voltage.

From equation (2), it is possible to estimate the \( V_{ds_{off}} \) of the switching transistor by measuring the input voltage, the overall period T and the time Ton. The first two parameters are easily measured from the converter terminals, while Ton can be measured from the output ripple.

VICOR converters with nominal input voltage of 300V (V300B12C250A, V300B5C200A) and 375V (V375B5C200A, V375B12250A) use similar transistors and have been used for these tests. The input voltage selected to

![Fig. 3. Vds_off of the power transistor Q as function of the output current with the converter operating at reduced input voltage. a) Converter V300B5C200A, Vin = 200V; b) Converter V300B12C250AL, Vin = 200V; c) Converter V375B5C200A, Vin =260V; d) Converter V375B12C250AL Vin = 260V.](image-url)
operate both DC-DC converters are the minimum voltage suggested by the manufacturer plus an additional safety margin. When the input voltage is lower than that limit, the converter automatically shuts-off. The values calculated from equation (2) are shown in figure 3 which depicts the drain-source voltage $V_{ds,off}$ for different converters under different operating conditions. $V_{ds,off}$ clearly decreases when the input voltage, output voltage and output current are de-rated from their nominal values.

III. SEB, SEGR in Power MOSFET Transistors

Destructive SEB effects on n-channel power MOSFETs were first reported in 1986 by Waskiewicz et al. [9] after testing those power devices with heavy ions. Since then, extensive studies on SEB and SEGR have been conducted. A review paper by Titus and Wheatley presents a comprehensive bibliography on these topics [10]. At the beginning of the present studies, tests based on a non-destructive technique were performed using heavy ions to measure the SEB cross section. In parallel, extensive analysis and modeling of SEBs have been conducted to characterize the phenomena involved and to develop design techniques in order to produce SEB tolerant devices [11]-[13]. Studies with heavy ions also showed a non-negligible SEB cross section at low linear energy transfers (LET). Based on these measurements, it was possible to predict that Si recoils induced by high-energy protons and neutrons can cause SEB. Some reports of tests performed on high voltage power MOSFETs have shown SEB during irradiation with high-energy protons. Oberg et al. [6] and Normand et al. [5] reported SEB in n-MOSFET using high-energy neutrons. In general, tests performed with high energy neutrons and protons gave similar SEB cross section for the same device [5][6]. When compared to heavy ions, the SEB cross section for nucleons are several orders of magnitude lower.

Normally devices operating in an environment with high-energy neutrons or protons require a de-rating in the operating voltage to perform reliably. The higher the rated voltage of the device, the higher the fractional de-rating required. In general, p-channel MOSFETs are much less sensitive to burnout than equivalent n-channel devices [5].

The SEB mechanism is associated with a second breakdown of the parasitic bipolar transistor intrinsic to the power MOSFET structure [5]. Due to the extremely low LET of high-energy neutrons and protons, charge carrier multiplication is necessary to induce SEB. Si recoils, product of hadron-Si inelastic collisions, have LET of the order of 10-15 MeV.cm$^2$/mg and can induce SEB. The mechanism of this destructive event can be explained by the Kuboyama model [5][14]. For MOSFETs operating at low Vds, heavy ion strikes only induce current filaments due to the direct charge deposition of the ion. Increasing Vds causes two effects: 1) the avalanche in the reverse biased epitaxial region; 2) the activation of the parasitic transistor. These two effects induce a regenerative process and more electrons are injected by the transistor in the depletion region and more holes from the avalanche directly bias the parasitic transistor. This effect rapidly drives the parasitic bipolar transistor to breakdown.

The end result is a sudden collapse of the drain-source impedance and, if the current is not controlled by the external circuit, the MOSFET is destroyed.

SEGR in power MOSFETs was not recognized as a serious problem until manufacturers started developing SEB-hardened MOSFETs. Standard MOSFET devices are more susceptible to SEB effects than SEGR. The latter is a destructive effect that can be described as the result of the energy released through the insulator by a heavy ion strike when the gate is biased by a voltage higher than a critical value [10]. However, this type of effect has also been observed during proton irradiation [15].

IV. Irradiation Tests

Several irradiation tests have been performed on different samples of VICOR converters. When exposed to low-energy neutrons (mean energy around 0.75 MeV), the performance of the converter has shown no appreciable degradation up to a fluence of 10$^{12}$ neutrons/cm$^2$. This test was performed to explore possible displacement damage sensitivity of the control electronics, which is in bipolar technology. The total fluence achieved in the test is almost an order of magnitude higher than the one expected in the application and gives us confidence that displacement damage will not be a problem.

In order to analyze the robustness to SEEs and define a safe de-rating for the input voltage, output voltage and output current, we have performed a series of proton irradiation tests (protons with energy of 60, 200 and 300 MeV) on VICOR converters and individual components used in the converters. The maximum fluence during the tests for both individual converters and components was in all cases below the maximum total dose these commercial devices could tolerate without any appreciable change in their electrical characteristics.

The switching transistor Q in the DC-DC converters is a N-MOSFET power device. We were able to identify the power transistor actually used in the converter thanks to the collaboration of the manufacturer. Two different but electrically very similar power MOSFET (rated 600V/6A) are used as the switching transistor Q in DC-DC converters produced by VICOR. The converter can include any of these transistors, produced by different manufacturers and to which we will refer as Q1 and Q2.

A. Non-destructive SEB tests

A 60 MeV proton beam has been used to measure the SEB cross section of the transistors as a function of the applied drain-source voltage with the power transistor in OFF state. During this non-destructive test, transistors are biased at different voltages through a protection resistor connected to the drain. The gate-source terminals were short-circuited keeping $V_{gs} = 0V$ during the test. Current spikes due to SEB induced by the proton beam are measured and counted during the irradiation. Test cards, each containing four power MOSFETs were irradiated, keeping all devices biased at the same potential during the test. The cross section was calculated as the ratio between the average number of SEBs measured and the integrated fluence (1.0x10$^9$ protons/cm$^2$).
During tests, the proton flux was equal to 1.0x10^11 protons/(cm²·sec). Results of the radiation tests are depicted in figure 4. Switching transistor Q1 exhibits higher cross-section than transistor Q2. This difference can be attributed to the fact the transistors are produced by different manufacturers and the design and process can be different. Based on the foreseen accumulated fluence in 10 years of operation (1.7-3.4x10^13 neutrons/cm²) and the results shown in figure 4, one can predict that transistor Q1 operating at $V_{ds}=350$V will have, in average, 6.8x10^2 failures/10 years. This value is equal to 77.62 failures / 10^8 h, or a mean-time-to-failure (MTTF) = 12.8x10^8 h. The MTTF value can even be further increased operating Q1 at $V_{ds}$ below 300V and, in the case of Q2, below 350V. Since it is unknown which one is mounted in each DC-DC converter, it is assumed for safety that the most sensitive one (Q1) is used.

![Cross section curve for Q1 and Q2](image)

Fig. 4. 60MeV protons cross section for Q1 (open symbols) and Q2 (closed symbols) power MOSFETs, both rated 600V/6A. Error-bars depict the minimum and maximum cross sections

The cross section curve of the sensitive device allows estimating the minimum $V_{ds_{off}}$ to apply to the power MOSFETs. As depicted in figure 3, this $V_{ds_{off}}$ is related to the operating conditions of the DC-DC converter. Combining these results with the estimated $V_{ds_{off}}$, it is possible to define de-rating factors for the input voltage, output voltage and output current of the DC-DC converter to allow reliable operation of the unit under high-energy neutron radiation. The main advantage of this procedure resides in the possibility to estimate the de-rating factors based on a careful analysis of the converter and a non-destructive test of the critical devices.

Assuming a limiting $V_{ds_{off}}$ of about 300V for the switching transistor Q, from figure 3, it is possible to predict that the converter V300B5C200A ($V_{in}=300$V / $V_{out}=5$V) can operate reliably in the described environment if the input voltage is de-rated to 200V and the output current is limited to 25A. In this limit condition the transistor Q will operate at $V_{ds_{off}}=300$V, with a MTTF better than 12.8x10^8 h. Following a similar procedure, the converter V300B12C250AL ($V_{in}=300$V / $V_{out}=12$V) will operate reliably if the input voltage is de-rated to 200V and the current is lower than the maximum value. De-rating the output voltage of this converter to 7.7V, figure 3 shows the output current can be increased up to the maximum value without the transistor Q reaching the limit $V_{ds_{off}}=300$V. The converter V375B5C200A ($V_{in}=375$V / $V_{out}=5$V) can operate in our environment if the input is de-rated to 260V and the output current is limited to less than 10A, while the converter V375B12C250AL ($V_{in}=375$V / $V_{out}=12$V) can operate under similar conditions if the $V_{in}=260$V and the output current is lower than 5A. From this analysis, one concludes that de-rating the group of converters with 300V nominal input voltage is a valid option for our application, while the group of converters with 375V nominal input voltage is not an adequate solution for our application because it is necessary to de-rate them to unpractical levels.

### B. Destructive tests

Several destructive tests on the complete DC-DC converters have been performed to verify our conclusion reported in Subsection IV.A. During the tests, the input voltage and the output current were continuously monitored and the temperature of the converter and heat sink was kept at about 40-45°C using forced air. This is the expected working temperature in the final design. Since the proton beam was larger than the whole converter, we could actually test the complete system, including the control circuitry, at the same time as the power transistor. The control circuit did not exhibit latch-ups, destructive misfiring, etc.; in fact the complete converter proved to be robust under high-energy proton irradiation.

Table II describes the results obtained using 60MeV, 200MeV and 300MeV proton beams. Converter failures were traced back to SEB of the switching transistor. $V_{ds}$ conditions for the switching transistor are specified for each test. In cases of failure, the fluence specified in the table corresponds to the occurrence of the SEB. The maximum integrated fluence was limited to avoid inducing total dose effects on these units, except during the test of the converter V375B5C200A with 300MeV protons. In this case, one unit was tested for increasing output currents to verify in more detail the assumptions made on Subsection IV.A. At each current value, the maximum fluence was limited to 0.5-1.0x10^11 protons/cm² such that the total fluence during the test did not exceed the maximum level of 3.0x10^11 protons/cm².

The reported data shows that converters can operate safely in high-energy proton environment up to a fluence of 1.0-3.0x10^11 protons/cm² if $V_{ds_{off}}$ is lower than 300V. As it was analyzed above, reducing the input voltage is not enough for safe operation. In addition to such de-rating, converters have to operate either at reduced output voltage (and nominal output current) or at reduced output current (and nominal output voltage).
For our application, de-rating both the input and output voltage and current is still a valid solution. Two converters are necessary to deliver to the front-end electronics 7.5V/20A and 5.2V/25A. Using the V300B12C250AL unit with an input voltage of 200V and an output voltage of 7.5V, table II shows that no failure occurred up to a fluence of $3.0 \times 10^{11}$ p/cm$^2$ when the converter operates at maximum output current. Similarly, no failures occurred up to a fluence of $2.0 \times 10^{11}$ p/cm$^2$ when the V300B5C200A operates with 200V input voltage, nominal output voltage and a maximum output current of 25A. De-rating the converter affects the efficiency but the penalty is tolerable: in both cases the efficiency decreases to about 76%, while in nominal conditions it is 82%.

C. Discussion of the results

Radiation tests are performed to evaluate the reliability of the unit tested operating under a new foreseen environment. A distinction needs to be drawn between the interpretation of tests which measure the change in characteristics of the device as function of the dose, and tests which induce catastrophic failures. Tests inducing displacement damage and total dose effects can be considered as a measurement of either the new life-time or variation of the principal characteristics for an estimated level of radiation. Test inducing destructive effects are more related with statistically random failures of a unit during its life-time. The results from table II only give a probabilistic measure of the future behavior of converters operating under a neutron environment. It is not possible to guarantee that if a particular sample tolerates a given fluence, another sample will work for the same fluence. The results in table II in conjunction with the SEB cross section measured for the power transistor can give a better indication of the unit reliability operating under the foreseen environment. The measurement of the transistor cross section allows a better definition of a threshold $V_{ds}$ voltage for more safe operation of the critical device. Furthermore, the measurement of the cross section of the critical device allows a better estimation of the new MTTF of such a device operating in a radiation environment. This is an important design parameter when the system is composed by a high number of converter units or there is limited access to repair.

V. Conclusions

This work has presented results of proton irradiation tests to validate the operation of VICOR converters in an environment with high-energy neutrons. We developed a methodology to predict the de-rating necessary for the input/output variables of the converter. This methodology is based on the analysis of the power converter to estimate the blocking-voltage across the critical devices and the measurement of the SEB cross-section of such devices. Further analysis and test are necessary to predict the reliability of a high number of converters in the foreseen environment, in particular their mean-time-to-failure. Our future work is oriented in that direction.

VI. Appendix I

This appendix gives a brief description of the mathematical analysis of the DC-DC converter and defines the relationship between $V_{ds,off}$ and the output conditions of the converter. For a deeper understanding of quasi-resonant converters the reader is referred to [16][17]. This study is mainly extracted from [16]. Based on the analysis presented for the zero-current-switched quasi-resonant converters, the inductor current $i_{lr}(t)$ can be expressed for $t$: $0 < t < T1$ as:

$$i_{lr}(t) = \frac{aV_{in}t}{Lr}$$

(α - 1)
where:

- \( L_r \): series parasitic inductance of the transformer referred to the secondary side.
- \( a = N_2/N_1 \): transformer turn ratio between primary and secondary.
- \( ilr(t) = a.I(t) \): current in the resonant inductor.

During this interval the diode D2, in figure 2, is ON carrying the load current and \( ilr(t) \), which grows from zero to \( I_{out} \). At the instant \( T_1 \), the diode D2 turns OFF and for \( T_1 < t < T_2 \), there is a resonance between \( L_r \) and \( C_r \). If we define, \( C_r \) the resonant capacitance, \( Z_n = \frac{L_r}{\sqrt{L_r C_r}} \): the characteristic impedance of the resonant circuit and \( \omega = 1/\sqrt{L_r C_r} \): the resonant angular frequency; the current follows:

\[
ilr(t) = I_{out} + \frac{a.Vin}{Z_n} \cos(\omega t) \quad (a-2)
\]

up to the instant \( t = T_2 \), when \( ilr(t) = 0 \). For \( t > T_2 \) the inductor current remains equal to zero as depicted in figure 2 with \( T_{on} \equiv T_2 \).

The resonant capacitor voltage \( V_{cr}(t) \) for \( 0 < t < T_1 \) is equal to zero due to the diode D2 being ON. When D2 turns OFF, for \( T_1 < t < T_2 \), \( V_{cr}(t) \) is:

\[
V_{cr}(t) = a.Vin.(1-\cos(\omega t)) \quad (a-3)
\]

At \( t=T_2 \), the inductor current is null and the resonant capacitor is discharged to zero by the load current before the next switching cycle starts. Then for \( t: T_2 < t < T_3 < T \), \( V_{cr}(t) \) is:

\[
V_{cr}(t) = a.Vin.(1-\cos(\omega T_2)) - I_{out}.t/Cr \quad (a-4)
\]

The energy is transferred from primary to secondary during \( T_{on} \equiv T_2 \). This input energy \( E_{in} \) can be written as:

\[
E_{in} = a.Vin.\int_{0}^{T_1} ilr(t).dt + \int_{T_1}^{T_2} ilr(t).dt \quad (a-5)
\]

after \( T_2 \equiv T_{on} \), the inductor current \( ilr(t) \) is equal to zero and the energy stored into the resonant capacitor is transferred to the load during the interval \( T_3 - T_2 < T_{off} \). The output energy per period is

\[
E_{out} = V_{out}.I_{out}.T \quad (a-6)
\]

Equating the input and output energy per period and using the equations (a-1) to (a-4), the output voltage is:

\[
V_{out} = \frac{a.Vin}{T} \left[ \frac{T_1}{2} + (T_2 - T_1) + (T_3 - T_2) \right] \quad (a-7)
\]

Given \( I_{out}, T \) and \( Vin \), equation (a-7) can be solved as ([16], eq (8)):

\[
\begin{align*}
V_{out} &= \frac{fs}{\omega} \left[ \frac{I_{out}.Zn}{2.a.Vin} + \alpha + \frac{a.Vin}{Zn.I_{out}} \right] \\
&\quad \left[ 1 + \sqrt{1 - \left( \frac{I_{out}.Zn}{a.Vin} \right)^2} \right] \quad (a-8)
\end{align*}
\]

where \( \alpha = \arcsin\left(\frac{I_{out}.Zn}{a.Vin}\right) \) with \( \pi < \alpha < 3\pi/2 \) and \( fs = 1/T \) is the switching frequency.

The time \( T_{on} \) is approximately constant and can be calculated from (a-1) and (a-2) as:

\[
T_{on} = \frac{1}{\omega} \left[ \pi + \frac{I_{out}.Zn}{a.Vin} + \arcsin\left( \frac{I_{out}.Zn}{a.Vin} \right) \right] \quad (a-9)
\]

Combining equations (2), (a-8) and (a-9), it is possible to define the relation between \( V_{dsoff} \) and the external variables \( V_{out} \) and \( I_{out} \). Figure A1 depicts the variation of \( V_{dsoff} \) of the power transistor \( Q \) as function of the output current for a constant output voltage, while figure A-2 shows \( V_{dsoff} \) for different output voltages when the output current is held constant. The parameter used to calculate both figures are:

\[
Vin = 300V; \quad a = 1/30; \quad \omega = 2.\pi \cdot 1MHz; \quad a.Vin / Zn = 60A.
\]

![Fig. A1. \( V_{dsoff} \) as function of \( I_{out} \) for \( V_{out} = 5V \), calculated from eqs (2), (a-8) and (a-9).](image1)

![Fig. A2. \( V_{dsoff} \) as function of \( V_{out} \) for \( I_{out} = 20A \), calculated from eqs (2), (a-8) and (a-9).](image2)
VII. ACKNOWLEDGMENTS

The authors would like to thank to CERN RD-49 program and CMS EMU/HCAL collaboration for financial support, to CERN EP-ESS group for support during the preparation of prototypes and for providing the necessary instrumentation for conducting those tests. Also, they would like to express the gratitude to the personnel of the three facilities used to perform the tests, the Cyclotron Research Center, Louvain-la-Neuve, Belgium; the Paul Scherrer Institute, Villigen, Switzerland and the Indiana University Cyclotron Facility (IUCF), Indiana, USA.

One of us (C.R.) thanks to A. Ronzhin and S. Los from Fermilab for performing the last radiation test at IUCF and to the PIXEL-BTeV group for providing the time slot from its schedule to perform the irradiation.

VIII. REFERENCES

[1] “A global radiation test plan for CMS electronics in HCAL, Muons and Experimental Hall”
http://cmsdoc.cern.ch/~faccio/proced.pdf